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Mechanics of shear zones in isotropic dilatant materials

M. CASEY*

Department of Earth Sciences, The University, Leeds LS2 9JT, England

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Abstract—For the present paper it is assumed that rocks have linear isotropic viscous or elastic deformational properties, but their moduli may vary with mean stress. It is argued that the rock in shear zones should be weaker than the surrounding rock. Models of shear zones in which they are represented as sheets of rock with lower elasticity or viscosity modulus are set up. The stresses within the shear zones are calculated for different orientations of the shear zones relative to the regional compression direction. For brittle failure it is found that the modification of the stress in the shear zone can favour further cracking and that the deformation is unstable. The model is used to provide an explanation for brittle failure in shear, second-order faults and en échelon arrays of tension fractures. The mean stress in a weak zone is different from that in the matrix, and it is proposed that shear zones in rocks deformed at high metamorphic grade are caused by ductility enhanced as a result of reactions promoted by changes in mean stress.

INTRODUCTION

SHEAR zones are often observed in homogeneous, isotropic rock masses (Ramsay & Graham 1970). The development of shear instabilities in anisotropic materials may be understood in terms of the theory propounded by Biot (1965) and applied to geological deformations by Cobbold *et al.* (1971). The mechanics of shear instability in isotropic materials has, until recently, been largely unknown. Cobbold (1977) has demonstrated qualitatively that strain softening can cause an instability which grows into a shear zone. Poirier (1980) shows that shear instabilities can develop in materials with a power law dependence of strain rate upon stress. Sewell (1974) and Rudnicki & Rice (1975) have analysed the problem of localisation of deformation in shear bands in plastic materials, and have indicated the rheological properties which give rise to shear instabilities. In particular, Rudnicki & Rice (1975) demonstrate the importance of the dependence of deformational properties on pressure or mean stress. A simpler analysis of shear zones, based on isotropic linear elastic or viscous deformational properties, is presented below and the results of the analysis are used in an attempt to explain the development of shear zones in brittle, semi-brittle and ductile rock deformation.

SHEAR ZONE MODEL AND STRESS ANALYSIS

The basis of the argument presented below is the principle of minimum energy which states that, for elastic materials, when a body is subjected to stresses the deformation suffered by the body is that which gives the least stored elastic strain energy. For a homogeneous isotropic linear elastic body subjected to a boundary stress which is everywhere the same, the deformation will be such that strains and stresses are constant throughout the body. Any perturbation which does not affect the boundaries, such as an incipient shear zone, of the stress field from homogeneity will result in an increase in the total strain energy. Shear zones are observed to have formed in the interiors of approximately homogeneous rock masses. If the perturbation represented by the incipient shear zone is not to increase the total stored strain energy, the material in the shear zone must be weaker. For elastic materials this means that the Young's modulus of elasticity must be lower. Similar arguments hold for viscous deformation with the principle of minimisation of the rate of dissipation of energy in place of the principle of least stored strain energy, and the viscosity modulus in place of the Young's modulus.

For a two-dimensional analysis of stress in shear zones, these are represented as planar regions of infinite extent set in a matrix of material with greater Young's modulus of elasticity. The geometry is shown in Fig. 1. The most compressive principal stress in the matrix acts at an angle of φ to the edge of the weak zone. To calculate the stresses within the weak band when the stresses and strains in the matrix are known, a coordinate system with axes parallel and perpendicular to the zone boundary is set up. The matrix stresses are: σ_x, σ_y and τ_{xy}; the matrix strains are e_x, e_y and γ_{xy}; the layer stresses are σ'_x, σ'_y and τ'_{xy}; and the layer strains are e'_x, e'_y and γ'_{xy}. Tensile stresses are given positive values

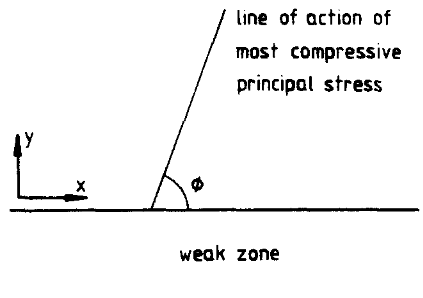

Fig. 1. The geometry of a shear zone represented as an infinitely long sheet of weaker material.

*Now at: Geologisches Institut, ETH-Zentrum, CH-8092, Zürich, Switzerland.

which is opposite to the usual geological convention, but the most compressive principal stress is given the usual symbol, σ_1. The boundary conditions between the layer and the matrix are, assuming the boundary is cohesive:

$$\begin{aligned} \sigma_y &= \sigma'_y \\ \tau_{xy} &= \tau'_{xy} \\ e_x &= e'_x . \end{aligned} \quad (1)$$

These three conditions enable the stresses and strains in the layer to be calculated from those in the matrix.

For plane strain deformation and tensile stresses given a positive value, the stresses are related to the strains by the linear equations:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{Bmatrix} \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix} . \quad (2)$$

For isotropic materials, the matrix $\{D\}$ is given by Zienkiewicz (1971, p. 54):

$$\{D\} = \frac{E(1-\mu)}{(1+\nu)(1-2\nu)} \begin{Bmatrix} 1 & \nu/(1-\nu) & 0 \\ \nu/(1-\nu) & 1 & 0 \\ 0 & 0 & (1-2\nu)/2(1-\nu) \end{Bmatrix} \quad (3)$$

where E is Young's modulus and v is Poisson's ratio.

The values of the principal stresses in the matrix are prescribed and for each value of φ it is possible to calculate the stresses in the x, y reference system (Fig. 1) by the standard equations for the transformation of stresses (Price 1966, chapter 1, equations 6 & 7). The strains can be calculated from the stresses by the inverse of equation (2). Two components of stresses in the layer, σ'_y and τ'_{xy}, are obtained directly from the boundary conditions (1); σ'_x may be found from the first two of equations (2) applied to the weak zone:

$$\begin{aligned} \sigma'_x &= d_{11}e'_x + d_{12}e'_y \\ \sigma'_y &= d_{21}e'_x + d_{22}e'_y . \end{aligned} \quad (4)$$

These may be solved for σ'_x to give:

$$\sigma'_x = d_{11}e_x + \frac{d_{12}}{d_{22}}(\sigma_y - d_{21}e_x) \quad (5)$$

where the primed terms on the right hand side have been replaced by their unprimed equivalents using the boundary conditions (1). Values of the principal stresses and their orientations can be calculated using standard equations (Jaeger 1969, p. 10, equation 19, & p. 7, equation 11).

Thus for any given stress state in the matrix, the state of stress in the layer may be calculated. A computer program was written to calculate the stress in the weak zone as a function of the angle between the zone and the most compressive stress, σ_1, in the matrix. Representive results are presented in the next section.

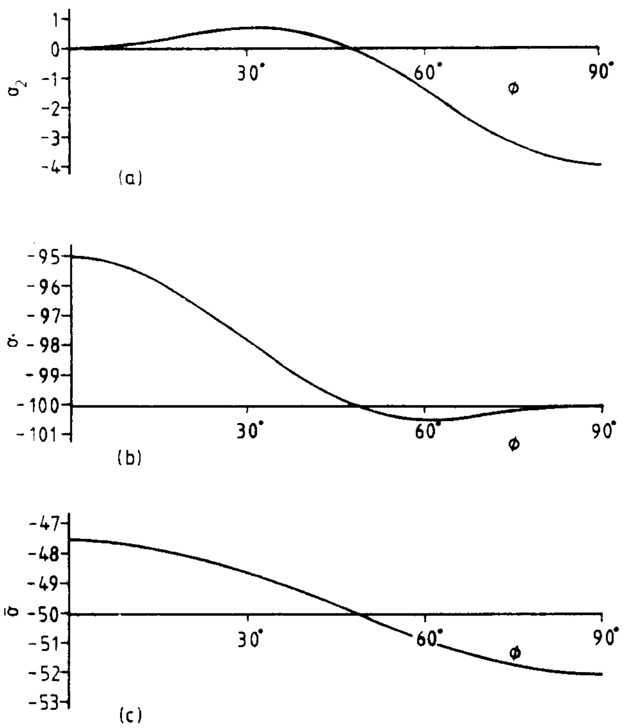

Fig. 2. Stress components σ_2 (a), σ_1 (b) and $\bar{\sigma}$ (c) in a weak zone, as functions of the angle φ between the line of action of σ_1 in the matrix and the boundaries of the zone. Stresses are shown in dimensionless units. Young's modulus in the weak zone is 95% of that in the matrix. Poisson's ratio is 0.45 throughout.

RESULTS

The variation of the stress components in the weak zone with the angle, φ, between the compressive stress direction in the matrix and the weak zone, are shown in Fig. 2 for a system with $\nu = 0.45$ throughout, and Young's modulus, E, in the layer equal to 95% of that in the matrix. At $\varphi = 45°$, the stress in the weak zone is almost the same as that in the matrix. Below 45° the mean stress is less compressive, and above 45° it is more compressive than it is in the matrix. The most tensile principal stress, σ_2, has a maximum at around $\varphi = 30°$, and σ_1 has a corresponding maximum in absolute value near 60°. Above $\varphi = 45°$, σ_2 falls below its value in the matrix. When the zone is very weak and $\varphi = 90°$ the stress in the zone is isotropic and has a value almost equal to σ_1 in the matrix. Similarly for a very weak zone with $\varphi = 0°$ the stress is isotropic with a value almost equal to σ_2 in the matrix. Varying the value of E in the weak layer does not change the positions of maxima in Fig. 2, but as the layer becomes weaker the stresses in the layer become more different from those in the matrix.

As the zone weakens the principal stresses in the zone rotate so as to be at 45° to the zone boundary for all values of φ except 0° and 90°.

From the Griffith theory of brittle failure the principal stresses must satisfy the following inequality if failure is to occur in shear:

$$\sigma_1 + 3\sigma_2 < 0 . \quad (6)$$

If this inequality is not satisfied the failure is in tension. It was found that stress states satisfying the condition for shear failure in the matrix could be modified in the weak zone so as to satisfy the condition for tensile failure. For this to occur it is necessary for σ_2 in the matrix to be zero or have a small positive value, and for the strength of the weak zone to be low.

APPLICATION TO GEOLOGICAL PROBLEMS

Brittle failure in shear

The Griffith theory of brittle failure and its modification by McClintock & Walsh (1962) predicts the stress conditions under which materials will fail in shear. It does not, however, show how the shear fracture develops. The propagation of individual microcracks occurs in a stable manner and cannot produce a fracture which runs through the whole body as in the case of tensile failure. Experiments have been carried out by Hallbauer *et al.* (1973) on the development of microcracks in deformed quartzite. Specimens obtained from the same piece of rock were compressed and the runs were stopped at different points on the stress–strain curve. The specimens were carefully examined and the location of the microcracks was noted. It was found that at low stresses the cracks occurred randomly throughout the specimen but as the ultimate strength was approached the cracks occurred in a planar zone oriented at an angle to the compression axis. In other words, a shear instability developed in which crack density became progressively higher until the rock disintegrated and a shear fracture plane developed.

The results of the stress analysis presented above can be used to explain the development of this shear instability. The Young's modulus of a material containing cracks falls with increasing crack density (Walsh 1965), and so the zone of increased crack density can be approximated by a plane of weaker material. For an instability to develop it is necessary that the modification to the stress in the weak zone should promote further

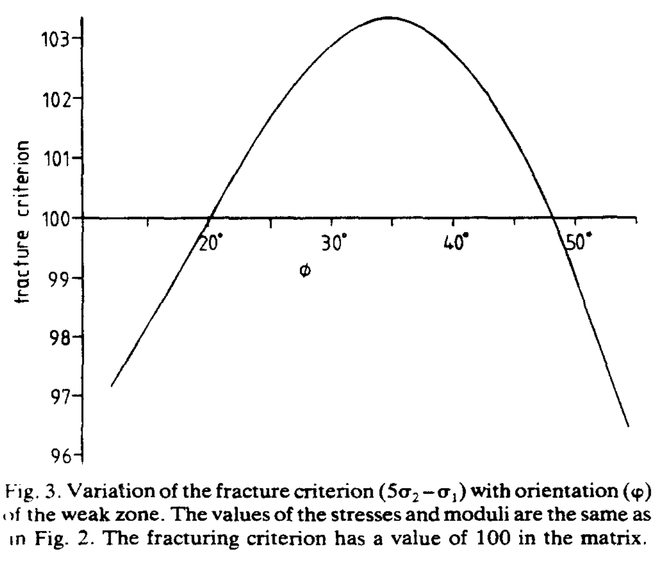

Fig. 3. Variation of the fracture criterion ($5\sigma_2 - \sigma_1$) with orientation (φ) of the weak zone. The values of the stresses and moduli are the same as in Fig. 2. The fracturing criterion has a value of 100 in the matrix.

cracking. The modified Griffith theory of McClintock & Walsh (1962) gives the following criterion for the onset of brittle failure:

$$\left\{\frac{(1 + \mu^2)^{1/2} + \mu}{(1 + \mu^2)^{1/2} - \mu}\right\} \sigma_2 - \sigma_1 = \text{constant} \qquad (7)$$

where μ is the coefficient of friction on the crack surface. Beniawski (1967) found a value of about 5 for the term in curly brackets in equation (7). The variation of the fracturing criterion, ($5\sigma_2 - \sigma_1$), with the angle of the weak zone to the compression direction is shown in Fig. 3. The unweakened material has a value of 100 for the fracturing criterion and thus weak zones orientated between 47° and 20° to the compression direction have stress conditions more favourable to fracture propagation than those in the matrix. The maximum of the fracturing criterion occurs at 35°, predicting conjugate shear planes at 70° to one another, rather than the commonly observed 60°. All variations of Poisson's ratio and stress in the matrix give values of more than 30° for the orientation of the least stable weak zone. This may be caused by several factors such as the neglect of plastic components in the deformation, or the assumption of isotropic properties for the cracked material.

Thus the analysis presented here shows how a shear instability may be initiated in brittle materials. The shear instability may develop by rapidly proceeding to runaway instability, producing a shear failure plane, or runaway instability may be retarded. The tendency of the mean stress within the weak zone to become less compressive results in an increase in pore volume. When the rock is saturated with fluid and the pore fluid pressure is important in initiating fractures, the increase in pore volume will cause a drop in pore pressure and tend to reduce crack propagation. The rate of development of the instability is then controlled by the rate at which fluid can flow in from the surrounding rock. This is in turn governed by the pressure gradient, the permeability of the rock and the distance over which fluid flow must take place. As a shear zone develops, the crack density increases, causing an increase in permeability. This will favour runaway instability, the zone which suffers further weakening becoming narrower, until a shear fracture develops.

Dilatancy is known to be a precursor of earthquake faulting (Scholtz *et al.* 1973). Sibson *et al.* (1975) have considered the effects of dilatancy development and its subsequent collapse after an earthquake on the pore fluid pressure and fluid movement near earthquake faults. The fact that dilatancy is recovered after failure indicates that much of the deformation associated with the development of the shear instability is elastic. This increases confidence in the explanation of shear instability in terms of elastic deformational properties as presented here.

The model of faulting given above may be used to explain the occurrence of second-order faults. The geometry of second-order faults is shown in Fig. 4. The angles which the second-order faults make with the main fault are 15° and 75° in Fig. 4. Moody & Hill (1956) have

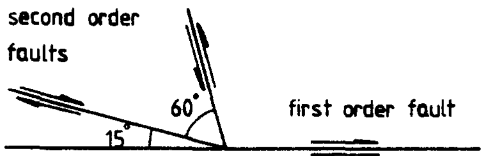

Fig. 4. Geometry of second-order faulting.

different angles, but most other authors (e.g. McKinstry 1953, Chinnery 1966, Arthaud & Matte 1979) give the orientations shown in Fig. 4. It is proposed that the main fault occurs in the middle of a weakened zone. In this zone the principal stress directions are close to 45° to the zone boundary and the second-order zones develop in response to the new stress direction. If the weak zones form at 30° to the σ_1 direction, then the orientation and sense of shear on the resulting faults are as in Fig. 4. This explanation has much in common with those of Anderson (1951) and McKinstry (1953) but the means by which the stress orientation changes are different.

In many regions of the crust deformed in the brittle or semi-brittle field, there are en échelon arrays of quartz- or calcite-filled tensile fractures. The geometry of this feature is shown schematically in Fig. 5. Each array of fractures defines a shear zone and the angle between the shear zone and the inferred compression direction is usually about 30°. This angle is the same as that for brittle failure in shear conditions. The tensile fractures appear to have been formed by tensile stress inclined at 45° to the shear zone boundary. The stress conditions for failure in shear or tension are given in equation (6) above. If both the initiation of the shear zone and the tensile fractures are the result of brittle failure, the stress in the rock at the outset must satisfy equation (6), and the stress within the zone must be modified so as to no longer satisfy equation (6). As described above, the stress in the weakened zone can be modified so as to satisfy the criterion for tensile failure if the effective least compressive principal stress is zero or positive.

Thus the development of shear zones with en échelon tension fractures can be understood in terms of the mechanism presented above for brittle failure in shear. The effective stress before the onset of shear instability has the least compressive stress close to zero. As the shear instability develops, it is prevented from achieving runaway instability. As the Young's modulus of the weak zone falls, the axes of the principal stresses rotate so as to be at 45° to the zone boundary and take on values which allow tensile fracture. The tensile fractures then fill with pressure-soluble material. This explanation for the initiation of the shear zones is in agreement with the conclusions of Knipe & White (1979) who report microstructural evidence for microcracking in the early stages of the deformation.

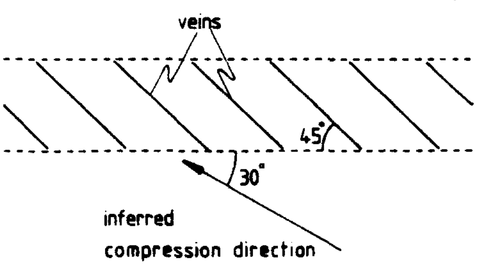

Fig. 5. Geometry of shear zones consisting of en échelon tension gashes.

For this structure, runaway instability may be prevented by reduction of pore pressure, as suggested above, or by partial sealing of the microcracks (Ramsay in press, Knipe & White 1979). If the sealing of microcracks is important in the development of these structures it will cause the induced strains in the rock to be non-recoverable and in this case a strictly elastic analysis is not applicable. A complete finite element modelling of the deformation would provide a useful insight into how these structures form.

Thus the purely elastic analysis presented in this paper can be used to understand several features of brittle failure and gives results broadly in agreement with the plastic analysis of Rudnicki & Rice (1975). The fact that dilatancy around earthquake faults is recoverable suggests that elasticity has a part to play in the development of shear instabilities. Combination of both elastic and plastic effects is probably necessary if a complete analysis is required.

Ductile zones

Shear zones are observed in rocks deformed at high metamorphic grade. Where conjugate shear zones are observed, the angle between the shear zones facing the compression direction is often greater than 90° (Ramsay 1980). Large, vertical shear zones with wrench-fault displacements have been observed in Greenland by Watterson (1978) and in Tibet by Tapponnier & Molnar (1977). Although conjugate sets were not observed by these authors, there are shear zones of opposite sense of shear which are related to the same compression direction. The angle between the shear zones and the compression direction is greater than 45°.

A possible mechanism of deformation for shear zones in metamorphic rocks is transformation or reaction enhanced ductility. These mechanisms are reviewed by White & Knipe (1978). In these modes of deformation, phase transformations or metamorphic reactions, respectively, cause a lowering of the viscosity of the rock.

The stability of phases or mineral assemblages is determined by, among other things, the temperature and pressure conditions as shown in Fig. 6. For

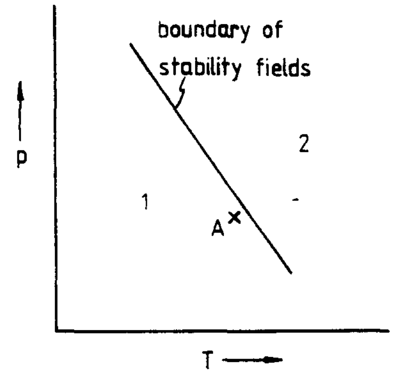

Fig. 6. Phase stability as a function of pressure, p, and temperature, *T*.

conditions of non-hydrostatic stress the mean stress is usually considered.

If a rock is subjected to a differential stress under conditions of mean stress and temperature which are close to a boundary of a stability field (point A in Fig. 6) it is possible that small heterogeneities will result in regions of the rock in which conditions lie on, or beyond, the boundary. This will initiate the phase change and weaken the rock in the region. If any of the regions have an elongate shape with the long dimension at more than 45° to the regional σ_1 direction, the weakening caused by the phase change will result in an increase in mean stress which will further promote the phase change.

For a change to a phase stable at lower pressure, shear instabilities would develop at angles of less than 45° to the regional σ_1 direction.

It can be seen from Fig. 2 that the greatest change in mean stress occurs when the weak zone is parallel or perpendicular to the regional σ_1 direction. When there is no shear stress on the boundary of the weak zone, a shear zone cannot develop and so the explanation presented above cannot predict the precise angle between the regional σ_1 direction and the shear zone. It is possible that transformation and reaction enhanced ductility give anisotropic deformation properties. Phase stability in non-hydrostatic stress conditions may be determined by a more complex function of principal stresses than the mean stress. The effects of these two factors could result in prediction of the angle of the shear to the compression direction. It would be necessary to identify the deformation mechanism and to develop a theoretical model of the deformation to incorporate other factors into the theory.

Thus the simple theory presented here shows how shear zones may develop in metamorphic rocks. Although it predicts that shear zones caused by transformations to phases stable at higher mean stress will be at angles greater than 45° to the regional σ_1 direction and those caused by transformations to lower mean stress phases will be at less than 45°, the precise angle is not predicted.

DISCUSSION

The idea that instabilities resulting in shear zones are caused by the dependence of the deformational properties of the rock on pressure or mean stress enables models to be set up which predict several features of shear zones observed in the field. The models presented above, based on linear, isotropic elastic or viscous properties, have some shortcomings. For brittle failure in shear, the angle between the shear zone and the compression direction is always greater than 30°. For shear zones in metamorphic rocks the model does not predict a specific value for the angle between the shear zone and the compression direction. Finite element analysis could be used to set up more realistic models and explore their consequences. In particular, the effects of some components of plastic strain could be investigated and the relationship of the model presented here to the theory of Rudnicki & Rice (1975) could be explored.

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